Social Choice Problem Set Not Required

It's OK to work together on problem sets.

Problems 1, and 2 deal with a Bergson-Samuelson social welfare function. For the economy consisting of the set H of households, a Bergson-Samuelson social welfare function can be described as

$$W(u^1(x^1), u^2(x^2), ..., u^{\#H}(x^{\#H}))$$

where u^i is household i's utility function and x^i is N-dimensional consumption vector. We assume

Positive association : $W_i > 0$ (the marginal social welfare of each household's utility is strictly positive; the subscript indicates a partial derivative)

Strict monotonicity: $u^i_k > 0$ (positive marginal utility for each good)

and assume an interior solution (just to make taking derivatives easy).

Consider a pure exchange economy, so that the resource constraint is

(1) $\sum_{i \in H} x^i = r$ where x^i and r are both positive N-dimensional vectors.

1. Show that any maximum of W subject to (1) is a Pareto efficient allocation (there are at least two ways to do this: you can develop the first order conditions to maximize W and show that they characterize a Pareto efficient allocation or go straight to the definitions).

2. Consider W with linear weights:

$$W(u^1(x^1), u^2(x^2), ..., u^{\#H}(x^{\#H})) = \sum_{i \in H} a^i u^i(x^i)$$

where $a^i > 0$, is a real number. Demonstrate the following result.

Proposition: Let x^{o1}, x^{o2}, ..., $x^{o\#H}$, be a Pareto efficient allocation subject to (1). Then there is a choice of a^i, $i \in H$, so that x^{o1}, x^{o2}, ..., $x^{o\#H}$, is a maximum of W subject to (1).

Does the proposition mean that a family of Bergson-Samuelson social welfare functions can determine as their maxima the whole range of Pareto efficient points?

Problems 3, 4, 5, 6 deal with the Arrow Possibility Theorem. Eliminating any one of the four restrictions (non-dictatorship, independence, Pareto principle, unrestricted domain) on an Arrow Social Welfare function allows us to find a successful Arrow SWF. That is, for any three of the four conditions, there is an Arrow Social Welfare Function that can fulfill those three. Demonstrate this result by finding a suitable Arrow SWF for each of the four sets of three conditions.

3. Find an Arrow SWF fulfilling non-dictatorship, independence, Pareto principle, but not unrestricted domain.

4. Find an Arrow SWF fulfilling non-dictatorship, independence, unrestricted domain, but not Pareto principle.

5. Find an Arrow SWF fulfilling non-dictatorship, Pareto principle, unrestricted domain, but not independence of irrelevant alternatives.

6. Find an Arrow SWF fulfilling independence, Pareto principle, unrestricted domain, but not non-dictatorship.